

Resonance phenomena in asymmetric superconducting quantum interference devices

T. P. Polak, E. Sarnelli

*Consiglio Nazionale delle Ricerche - Istituto Nazionale per la Fisica della Materia,
Complesso Universitario Monte S. Angelo, 80126 Naples, Italy and
Istituto di Cibernetica "E. Caianiello" del CNR, Via Campi Flegrei 34, I-80078 Pozzuoli, Italy.*

Theory of self induced resonances in asymmetric two-junction interferometer device is presented. In real devices it is impossible to have an ideal interferometer free of imperfections. Thus, we extended previous theoretical approaches introducing a model which contains several asymmetries: Josephson current ϵ , capacitances χ and dissipation ρ presented in an equivalent circuit. Moreover, non conventional symmetry of the order parameter in high temperature superconducting quantum interference devices forced us to include phase asymmetries. Therefore, the model has been extended to the case of π -shift interferometers, where a phase shift is present in one of the junctions.

PACS numbers: 74.72.-h, 74.50.+r

I. INTRODUCTION

Superconducting quantum interference devices (SQUIDS) are the most employed superconductive electronic circuits in practical applications.^{1,2,3,4,5,6,7,8,9} With the discovery of high-temperature superconductors (HTS) also high-temperature SQUIDS have been developed.^{10,11,12,13,14} This class of devices, although less sensitive than the most competitive low-temperature SQUIDS, have been used in several applications, where portability and/or positioning as much as high working temperatures are needed. Moreover, the demonstration of an unconventional symmetry of the order parameter in *YBaCuO* (YBCO),^{15,16,17} opened new horizons for using the so-called π -SQUIDS in superconductive electronics. Indeed, π -SQUIDS¹⁸ can be used to self-frustrate quantum bit circuits or to feed RSFQ (rapid single flux quantum) devices.^{19,20} As a consequence, a full knowledge of properties of HTS SQUIDS is at great importance. In particular, the aspects limiting their utilization in applications have to be explored. We can consider two effects limiting performance of HTS zero-or π -SQUIDS (zero indicates the conventional SQUID where no phase shift has been established along the superconducting loop): asymmetries in the junction properties and anomalous electrical behavior induced by an arbitrary phase shift in one of the two junctions forming the interferometer. Asymmetries in conventional low-temperature devices have been first examined by Tesche and Clarke.²¹ In their paper a complete study of the performance in terms of noise characteristics has been carried out. The interest on asymmetric SQUIDS grew up again after the discovery of HTS. Indeed, the parameter spread in HTS SQUIDS is often so large that significant asymmetries arise. Hence, it is particularly hard to fabricate two identical HTS Josephson junctions, even though they are very close to each other on the chip. Performance of asymmetric SQUIDS have been analyzed by Testa and co-workers.^{22,23} From their papers it is evident that higher magnetic sensitivities are achieved when asymmetric SQUIDS are used. The

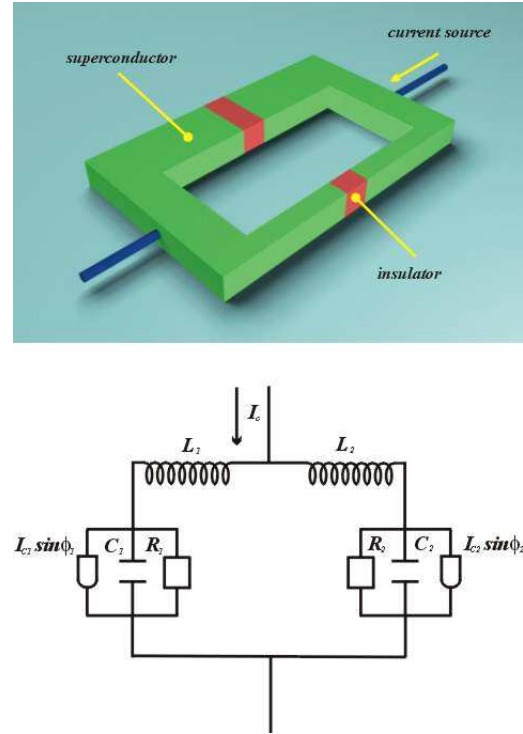


Figure 1: Theoretical model of an asymmetric superconducting quantum interference device and the equivalent circuit contains two Josephson junctions with the critical current I_{Ci} and parallel capacitance C_i . Each single junction contains parallel linear resistance R_i and the interferometer is fed by an external source I_c . The self-inductances of the junctions are equal L_i and ϕ_i is the phase difference across the i th junction.

asymmetry combined with a damping resistance leads to a flux to voltage transfer coefficient several times larger than the one typical for symmetric devices, together with a lower magnetic flux noise. The large ratio of the flux to voltage transfer coefficient allows a direct coupling to an external preamplifier without the need of an impedance matching flux transformer or additional positive feedback circuitry. This simplifies

the read-out electronics, as required in multi-channel systems for low-noise measurements. However, the final performance of a dc-SQUID is influenced by the presence of undesired anomalies occurring on the current-voltage (IV) characteristics, namely Fiske or resonant steps.²⁴ Such structures originate for the non-linear interaction between the resonant cavity, represented by the superconducting loop, and an rf current component - the ac Josephson current in the junctions. This system may be treated with the equivalent electrical resonant circuit, as shown in Fig. 1.

A deep investigation of the properties of resonances in asymmetric SQUIDS, also including different phase shift in the SQUID loop, is mandatory and can be very useful for people involved in SQUID design. Self-resonances occurring in superconducting interferometers are considered to be phenomena reducing performance of high-sensitive SQUIDS. Indeed, Zappe and Landman²⁵ first investigated experimentally resonances in low-Q Josephson interferometers. The analysis was taken again by Tuckerman and Magerlein,²⁶ who presented a theoretical and experimental investigation of resonances in symmetric devices. Successively, Faris and Walsamakis²⁷ showed characteristic of resonances in asymmetric two-junction interferometers, introducing an important distinction between current- and voltage-controlled cases. Based on their analysis, Camerlingo et al.²⁸ reported an experimental work showing the effect of the loop capacitance on resonant voltages in asymmetric interferometers. Recently, the nature of resonances in SQUIDS in which a significant flux is coupled to the Josephson junctions, called spatially distributed junctions (SDJ) dc-SQUID, has been analyzed by Chesca.²⁹ He showed that useful information about the order parameter symmetry can be provided by studying directly the magnetic field dependences of both the dc Josephson critical current and self-induced resonant modes of dc-SQUIDS made of non-conventional superconductors. The further analysis of voltage states in current-voltage characteristics of symmetric dc π -SQUIDS, in which the junctions are equal and not-distributed circuitual elements, has been done by Chesca and co-workers.³⁰ Moreover, d-wave induced zero-field resonances in dc π -SQUIDS have also been observed.³¹

In our work we present a full investigation of resonances in asymmetric SQUIDS, also in the presence of asymmetries in the junction phases. The outline of the paper is the following: In Sec. II we outline the model Hamiltonian, and we derive equations for asymmetric dc-SQUIDS. In Sec. III we present the method and assumptions which have been made. Sec. IV we present our results considering special cases and their relevance to the other theoretical works. Finally in Sec. V we discuss the relevance obtained results to the experimental situations.

II. MODEL

We start with defining an asymmetric superconducting quantum interference device (ASQUID) which consists of two Josephson junctions (see Fig. 1). Each of them has a critical current I_{C_i} and a parallel capacitance C_i . We assume also that single junction contains a parallel linear resistance R_i and interferometer is fed by an external source I_c , but the details of the equivalent circuit will be specified later. The self-inductances of the junctions in ASQUID are equal to L_1 and L_2 . We do not consider mutual inductances between the junctions. Hamiltonian of the ASQUID contains three parts^{24,32}:

$$\mathcal{H} = \mathcal{H}_C + \mathcal{H}_J + \mathcal{H}_M. \quad (1)$$

First term on the right side of Eq. (1) defines electrostatic energy

$$\mathcal{H}_C = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2, \quad (2)$$

where V_i is the voltage across the i th junction. The last equation can be transformed to the phase representation using the Josephson relation $\dot{\phi} = 2\pi/\Phi_0 V$:

$$\mathcal{H}_C = \frac{1}{2} \left(\frac{\Phi_0}{2\pi} \right)^2 (C_1\dot{\phi}_1^2 + C_2\dot{\phi}_2^2), \quad (3)$$

where ϕ_i is a phase difference across the i th junction. The second term is the Josephson energy:

$$\mathcal{H}_J = E_{J,1}(1 - \cos \phi_1) + E_{J,2}(1 - \cos \phi_2), \quad (4)$$

where $E_{J,i} = \Phi_0/2\pi I_{C,i}$. To complete the set of equations for the interferometer one should take into account that loop current I_L can contribute to the flux. The gauge invariant superconducting phase differences between the edges of any loop and magnetic flux are directly related by the fluxoid quantization relation:

$$\phi_2 - \phi_1 = 2\pi n + \phi_{ext} - \frac{2\pi}{\Phi_0} L_+ I_L, \quad (5)$$

where n is an integer and

$$L_+ = L_1 + L_2, \quad (6)$$

$$I_L = \frac{L_1 I_1 - L_2 I_2}{L_+}. \quad (7)$$

Finally, for $n = 0$ magnetic energy takes the form:

$$\mathcal{H}_M = \frac{1}{2} L_+ I_L^2 = \frac{1}{2} \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{(\phi_2 - \phi_1 - \phi_{ext})^2}{L_+}. \quad (8)$$

At this stage we do not provide an information about dissipative environment and external forces which will be discussed later. Applying the Euler-Lagrange equation

$$dt \left(\partial_{\dot{\phi}_n} \mathcal{L} \right) - \partial_{\phi_n} \mathcal{L} = 0 \quad (9)$$

to the Lagrangian

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \left(\frac{\Phi_0}{2\pi} \right)^2 (C_1 \dot{\phi}_1^2 + C_2 \dot{\phi}_2^2) \\ & - \frac{1}{2} \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{(\phi_2 - \phi_1 - \phi_{ext})^2}{L_+} \\ & - E_{J,1} (1 - \cos \phi_1) - E_{J,2} (1 - \cos \phi_2), \end{aligned} \quad (10)$$

we find equations of motion

$$\left(\frac{\Phi_0}{2\pi} \right)^2 C_1 \ddot{\phi}_1 + E_{J,1} \sin \phi_1 = \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{(\phi_2 - \phi_1)}{L_+}, \quad (11)$$

$$\left(\frac{\Phi_0}{2\pi} \right)^2 C_2 \ddot{\phi}_2 + E_{J,2} \sin \phi_2 = \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{(\phi_1 - \phi_2)}{L_+}. \quad (12)$$

Similarly to Tesche²¹ we introduce the following parameters

$$C_1 = (1 + \chi) C, \quad C_2 = (1 - \chi) C, \quad (13)$$

$$E_{J,1} = (1 + \epsilon) E_J, \quad E_{J,2} = (1 - \epsilon) E_J, \quad (14)$$

$$L_1 = (1 + \lambda) \frac{L}{2}, \quad L_2 = (1 - \lambda) \frac{L}{2}, \quad (15)$$

where dimensionless anisotropy quantities χ , ϵ and λ describe the relative deviations of the model parameters from the corresponding average values C , E_J and L . We can vary the values of the anisotropy parameters within the range $[0, 1]$, where zero leads to the isotropic model and value one completely rules out presence of one junction from the interferometer. Since $L_+ = L$ we conclude that a difference between inductances does not influence the dynamics of the model. After renormalization to dimensionless quantities

$$\omega_c^2 = \left(\frac{2\pi}{\Phi_0} \right)^2 \frac{E_J}{C} = \frac{1}{LC}, \quad (16)$$

$$\beta = \frac{2\pi}{\Phi_0} I_C L, \quad (17)$$

we finally obtain two coupled non-linear second-order differential equations describing an ASQUID:

$$(1 - \chi) \ddot{\phi}_1 + (1 - \epsilon) \sin(\phi_1 + \vartheta_1) = \frac{(\phi_2 - \phi_1)}{\beta}, \quad (18)$$

$$(1 - \chi) \ddot{\phi}_2 + (1 + \epsilon) \sin(\phi_2 + \vartheta_2) = \frac{(\phi_1 - \phi_2)}{\beta}. \quad (19)$$

Until now we have not considered dissipation effects and specific geometry of the circuit. First, we have to decide, what modes of operation we think about: current controlled (CC) or voltage controlled (VC)? This is a crucial point simply because a choice we make is going to affect our system. For the VC case where SQUID is excited by a voltage source V_s we have to add terms proportional to $V_s t$ to the equations. The difference caused

by various excitation sources affects frequencies of the oscillating modes of the system. In this paper we assume that SQUID is current excited by a constant current source (see Fig. 1). This foundation leads to an additional term γ_i in both equations. Origin of the last parameter is clear when we consider the equivalent loop of a real interferometer²⁶ where the center of the inductance is fed by a gate current source I_g . Using notation from Tuckerman's paper and the above information we can derive exact form of γ_i :

$$\gamma_1 = \frac{I_g + 2I_c}{2I_C}, \quad \gamma_2 = \frac{I_g - 2I_c}{2I_C}. \quad (20)$$

where I_c is a circulating current.

Considering dissipation due to a quasi-particle current we add parallel resistances R_i . These dissipative currents flowing through the junctions of the interferometer can vary from each other and, as a consequence we have to introduce their asymmetry assuming

$$(1 + \rho) \alpha = \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{1}{R_1}, \quad (1 - \rho) \alpha = \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{1}{R_2}. \quad (21)$$

Different phase shift can be added to each junction separately putting $\phi_i + \vartheta_i$ in Eq. (18) and Eq. (19). We see that values $\vartheta_0 = 0$ and $\vartheta_1 = \pi$ lead to the opposite sign of the current which means its opposite direction. Finally, we write the equations for ASQUID with phase shift in form:

$$\begin{aligned} (1 + \chi) \ddot{\phi}_1 + (1 + \rho) \alpha \dot{\phi}_1 + (1 + \epsilon) \sin(\phi_1 + \vartheta_1) \\ = \gamma_1 + \frac{(\phi_2 - \phi_1)}{\beta}, \end{aligned} \quad (22)$$

$$\begin{aligned} (1 - \chi) \ddot{\phi}_2 + (1 - \rho) \alpha \dot{\phi}_2 + (1 - \epsilon) \sin(\phi_2 + \vartheta_2) \\ = \gamma_2 + \frac{(\phi_1 - \phi_2)}{\beta}. \end{aligned} \quad (23)$$

In order to obtain similar node equations one can also use Kirchoff's current law to the specific circuit. We have to mention that the noise effects are not present in our analysis. Choice of parameters $\chi = \epsilon = \rho = \vartheta = 0$ stands for the fully symmetric case.

III. METHOD

We shall analyze two coupled differential equations (22) and (23) for the case $\beta \leq 1$ that coupling between the two junctions of the interferometer is strong and, hence the last terms of the right hand in Eqs. (22) and (23) play important role since they contain expressions proportional to $\pm \beta^{-1}(\phi_2 - \phi_1)$. Let us introduce new variables

$$\phi_- = \frac{\phi_1 - \phi_2}{2}, \quad \phi_+ = \frac{\phi_1 + \phi_2}{2}, \quad (24)$$

$$\gamma_- = \frac{\gamma_1 - \gamma_2}{2}, \quad \gamma_+ = \frac{\gamma_1 + \gamma_2}{2}, \quad (25)$$

$$\vartheta_- = \frac{\vartheta_1 - \vartheta_2}{2}, \quad \vartheta_+ = \frac{\vartheta_1 + \vartheta_2}{2}, \quad (26)$$

where ϕ_- represents the flux number (this parameter distinguishes interferometer from a point junction), and ϕ_+ is the average phase difference of the junctions. For the equivalent circuit of a real interferometer γ_- can be recognized as a control current I_c and γ_+ as a bias current I_g . Parameters ϑ_{\pm} are relative changes of the phase shifts present in each junction. In terms of the above we write equations (22) and (23) in form

$$\begin{aligned} \ddot{\phi}_+ + \alpha\dot{\phi}_+ + \sin(\phi_+ + \vartheta_+) \cos(\phi_- + \vartheta_-) - \gamma_+ \\ + \chi\ddot{\phi}_- + \alpha\rho\dot{\phi}_- + \epsilon \sin(\phi_- + \vartheta_-) \cos(\phi_+ + \vartheta_+) \\ = 0, \end{aligned} \quad (27)$$

$$\begin{aligned} \ddot{\phi}_- + \alpha\dot{\phi}_- + \sin(\phi_- + \vartheta_-) \cos(\phi_+ + \vartheta_+) - \gamma_- + \frac{2}{\beta}\phi_- \\ + \chi\ddot{\phi}_+ + \alpha\rho\dot{\phi}_+ + \epsilon \sin(\phi_+ + \vartheta_+) \cos(\phi_- + \vartheta_-) \\ = 0 \end{aligned} \quad (28)$$

In the following we have to assume a form of the solution. The voltage variations appearing in ASQUID come from the interaction between the junction current and circuit's elements. We assume voltage sinusoidal variations with dc component V , ac amplitude v , frequency ω and phase φ :

$$V(t) = V + v \cos(\omega t + \varphi), \quad (29)$$

where other harmonics are filtered out. Using the Josephson relations and integrating out we get for i th junction:

$$\phi_i(t) = \phi_{0,i} + \omega t \pm \delta \sin(\omega t + \varphi), \quad (30)$$

where $\delta = \frac{v}{\Phi_0}$. The flux number ϕ_- and the average phase difference ϕ_+ can be written:

$$\phi_- = \phi_c - \delta \sin \omega t, \quad (31)$$

$$\phi_+ = n\omega t - \theta. \quad (32)$$

where ϕ_c is the average value of the internal phase ϕ_- . In order to account the difference between odd and even behavior of the ASQUID interferometer we define:

$$\phi_- = \phi_c - \delta \sin \omega t - k \frac{\pi}{2}, \quad (33)$$

$$\phi_+ = n\omega t - \theta - k \frac{\pi}{2}, \quad (34)$$

where k is equal 0(1) for even (odd) number of resonances.

IV. RESULTS

Substituting expressions (33) and (34) into equations (27) and (28) and extracting by calculating average over time the dc, $\sin \omega t$ and $\cos \omega t$ Fourier components we get:

$$\begin{aligned} \alpha n \omega &= \gamma_+ - J_n(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-) \\ &+ \epsilon J_n(\delta) \sin(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-), \end{aligned} \quad (35)$$

$$\begin{aligned} \gamma_- &= \frac{2}{\beta} \phi_c - J_n(\delta) \sin(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-) \\ &+ \alpha \rho n \omega + \epsilon J_n(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-), \end{aligned} \quad (36)$$

$$\begin{aligned} -\chi \delta \omega^2 &= J_n^-(\delta) \cos(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-) \\ &+ \epsilon J_n^-(\delta) \sin(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-), \end{aligned} \quad (37)$$

$$\begin{aligned} \alpha \rho \delta \omega &= -J_n^+(\delta) \sin(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-) \\ &+ \epsilon J_n^+(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-), \end{aligned} \quad (38)$$

$$\begin{aligned} \delta \left(\frac{2}{\beta} - \omega^2 \right) &= J_n^-(\delta) \sin(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-) \\ &+ \epsilon J_n^-(\delta) \cos(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-), \end{aligned} \quad (39)$$

$$\begin{aligned} \alpha \delta \omega &= J_n^+(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-) \\ &- \epsilon J_n^+(\delta) \sin(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-), \end{aligned} \quad (40)$$

where

$$J_n^{\pm}(\delta) = J_{n-1}(\delta) \pm J_{n+1}(\delta) \quad (41)$$

and $J_n(\delta)$ is the Bessel function of the first kind.³³ Using Eq. (35), Eq. (40) and Bessel function identity, we obtain

$$\begin{aligned} \alpha n \omega &= \gamma_+ - \frac{J_n(\delta)}{J_n^+(\delta)} \alpha \delta \omega \\ &= \gamma_+ - \frac{\alpha \delta^2 \omega}{2n}. \end{aligned} \quad (42)$$

We define normalized excess current due to the resonance

$$I_{exc} = \frac{\alpha \delta^2 \omega}{2n}. \quad (43)$$

Above equations can be rewritten using the dimensionless damping parameter $\Gamma \equiv (\alpha \omega_r)^{-1}$, where ω_r is the resonant frequency. Gamma was introduced by Werthamer³⁴ and described the strength of the coupling of the current to the resonance in case of the junction coupled to cavity. Several authors used it as a damping parameter.^{25,26} We can combine equations (36) and (38) :

$$\gamma_- = \frac{2}{\beta} \phi_c + \alpha \rho n \omega + \frac{\alpha \rho \delta^2 \omega}{2n}, \quad (44)$$

and using relation for excess current we get

$$\gamma_- = \frac{2}{\beta} \phi_c + \rho \gamma_+. \quad (45)$$

We see that formula (43) derived for the excess current is universal in such sense that it holds even for asymmetric SQUID. This expression is also true in the presence of any changes of the phase shift in one of the junctions of

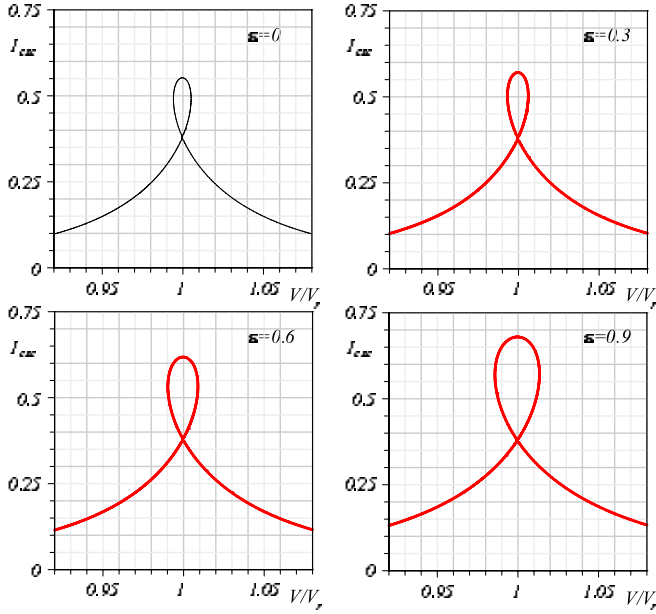


Figure 2: Current voltage characteristics with Josephson current anisotropy ϵ , first resonance, $\Gamma = 20$, $\beta = 0.1$. Black color of the curves used in this and next plots indicates the symmetric SQUID $\chi = \epsilon = \rho = 0$.

the interferometer. We can add squares of expressions (37-40):

$$\left[\frac{\delta(1 - \tilde{\omega}^2)}{J_n^-(\delta)} \right]^2 + \left[\frac{\alpha\delta\tilde{\omega}}{J_n^+(\delta)} \right]^2 + \left[\frac{\chi\delta\tilde{\omega}^2}{J_n^-(\delta)} \right]^2 + \left[\frac{\alpha\rho\delta\tilde{\omega}}{J_n^+(\delta)} \right]^2 = 1 + \epsilon^2, \quad (46)$$

where $\tilde{\omega} = \omega/\omega_r = V/V_r$ is the normalized voltage. From above and Eq. (43) we can derive normalized excess current dependence on voltage for given anisotropic parameters. However analysis is complex and is better to simplify our model considering special cases which could give us more insight into structure of resonances in ASQUID.

A. Special cases

For a general choice of parameters equations (35)-(40) are coupled and must be solved numerically. However considerations of special cases can provide more insights into general solution of the problem.

1. Asymmetry of the Josephson current ($\epsilon \neq 0$)

In that case we assume that only Josephson current asymmetry is present. Then the Eq. (46) can be reduced

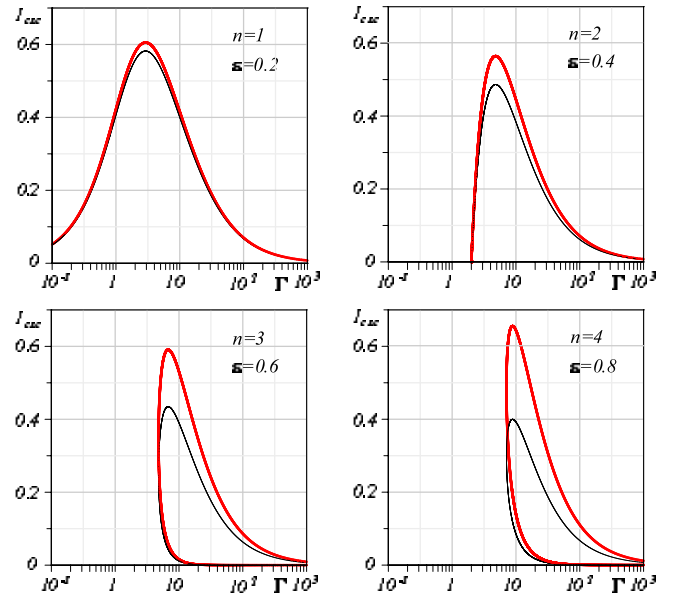


Figure 3: The normalized resonant current I_{exc} versus damping parameter Γ for different values of the Josephson current anisotropy parameters ϵ (red curves), n th resonance.

to form

$$\left[\frac{\delta(1 - \tilde{\omega}^2)}{J_n^-(\delta)} \right]^2 + \left[\frac{\alpha\delta\tilde{\omega}}{J_n^+(\delta)} \right]^2 = 1 + \epsilon^2. \quad (47)$$

From the above expression coupled with Eq. (43) we can derive the normalized current dependence on normalized voltage plots with ϵ asymmetry (see Fig. 2). Also the normalized resonant current versus damping parameter for several resonances can be obtained (see Fig. 3).

2. Asymmetry of the capacitances ($\chi \neq 0$) and resistances ($\rho \neq 0$) with phase shift ($\vartheta_{\pm} \neq 0$)

Let us consider case when $\epsilon = 0$ which means that asymmetry of the Josephson current is not present. In this case equations (35)-(40) are reduced to

$$\alpha n \omega = \gamma_+ - J_n(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-), \quad (48)$$

$$\gamma_- = \frac{2}{\beta} \phi_c - J_n(\delta) \sin(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-) + \alpha \rho n \omega, \quad (49)$$

$$-\chi \delta \omega^2 = J_n^-(\delta) \cos(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-), \quad (50)$$

$$\alpha \rho \delta \omega = -J_n^+(\delta) \sin(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-), \quad (51)$$

$$\delta \left(\frac{2}{\beta} - \omega^2 \right) = J_n^-(\delta) \sin(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-) \quad (52)$$

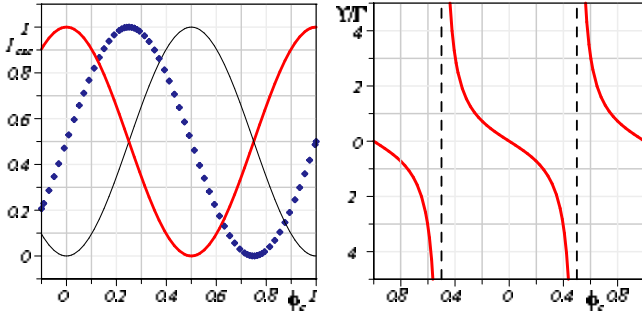


Figure 4: Dependence of the normalized resonant current I_{exc} with relative phase shift $\vartheta_- = 0.5$ (red), 0.25 (dot), 0 (black) and the ratio Y/Γ with no phase shift $\vartheta_- = 0$ versus magnetic field ϕ_c .

$$\alpha\delta\omega = J_n^+(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-). \quad (53)$$

First we will analyze low- Γ case in order to compare our results with the original calculations presented in literature.^{25,35}

Low- Γ devices with symmetric values of the Josephson current ($\epsilon = 0$)

At the resonance frequency $\omega = n\omega_r$ Eq. (52) is satisfied when $\theta = \vartheta_+$. This condition rules out equations with terms proportional to $\sin(\theta - \vartheta_+)$ and therefore there is no trace of the asymmetries of the Josephson current ϵ and dissipation ρ . For small gamma Γ devices $J_n^\pm(\delta) = 0$ for $n > 1$ and, hence only the first resonance exists. We can derive the following equations

$$-\frac{\delta}{\Gamma} = \cos(\phi_c + \vartheta_-), \quad (54)$$

$$\frac{\delta}{\Gamma} = \sin(\phi_c + \vartheta_-). \quad (55)$$

where $\Upsilon \equiv (\chi\omega^2)^{-1}$ is dimensionless parameter. Rearranging the last equation and putting into expression for excess current we get:

$$I_{exc} = \Gamma \sin^2(\phi_c + \vartheta_-), \quad (56)$$

which is general result for different SQUIDs. We can calculate other relations:

$$I_{exc} = \frac{\Upsilon^2}{\Gamma} \cos^2(\phi_c + \vartheta_-), \quad (57)$$

$$\frac{\Upsilon}{\Gamma} = -\tan(\phi_c + \vartheta_-). \quad (58)$$

which are plotted in Fig. 4. We see that the results obtained previously by other authors^{25,35} are presented in framework of our rather general calculations and can be derived as special cases.

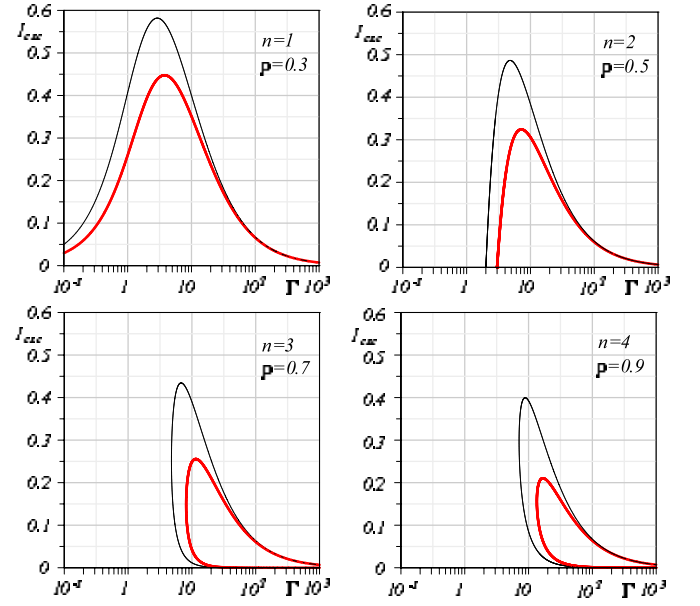


Figure 5: The normalized resonant current I_{exc} versus damping parameter Γ for different values of the dissipation anisotropy parameters ρ (red curves), n th resonance.

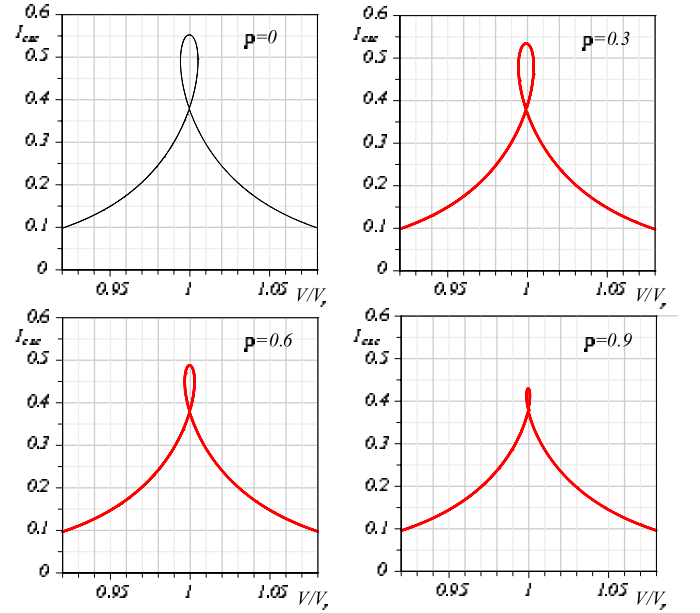


Figure 6: Current voltage ($I_{exc} - V/V_r$) characteristics with dissipation anisotropy ρ , first resonance ($n = 1$), $\Gamma = 20$, $\beta = 0.1$.

Analysis for not small Γ

When Γ is not small we cannot simplify equations using condition under which Bessel functions can be approximated by zero except the case of the first resonance.

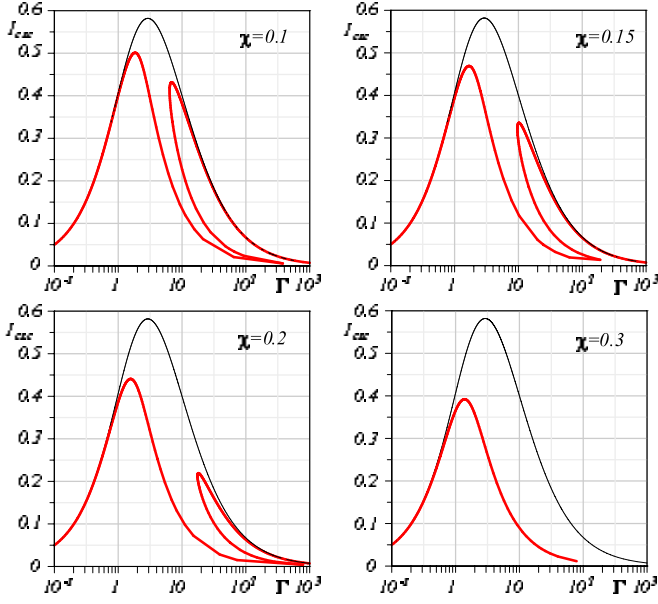


Figure 7: The normalized resonant current I_{exc} versus damping parameter Γ , different capacitance anisotropy parameter χ , first resonance ($n = 1$).

Putting $\epsilon = 0$ in Eq. (46) we obtain:

$$\left[\frac{\delta(1 - \tilde{\omega}^2)}{J_n^-(\delta)} \right]^2 + \left[\frac{\alpha\delta\tilde{\omega}}{J_n^+(\delta)} \right]^2 + \left[\frac{\chi\delta\tilde{\omega}^2}{J_n^-(\delta)} \right]^2 + \left[\frac{\alpha\rho\delta\tilde{\omega}}{J_n^+(\delta)} \right]^2 = 1. \quad (59)$$

From the above equation and expression (43) for the excess current we can derive the normalized current voltage characteristics for ASQUID. The second and the fourth terms of above equation can be combined. We see that influence of the anisotropy of the dissipative current

$$\left[\frac{\alpha\delta\tilde{\omega}}{J_n^+(\delta)} \right]^2 \rightarrow [1 + \rho] \left[\frac{\alpha\delta\tilde{\omega}}{J_n^+(\delta)} \right]^2 \quad (60)$$

manifests by the decreasing of the maximum value of the resonant current, for given n th resonance mode, when we increase the anisotropy parameter ρ (see Fig. 5 and Fig. 6). We observe a shift of the maximum value of I_{exc} toward higher values of the damping parameter Γ . Analysis of the influence of the anisotropy of the capacitances can be done in the same manner. We can again merge first and third terms of the Eq. (59). Contrary to previous simple case present one is more complex merely because we have taken into account element proportional to $\tilde{\omega}^4$ which produces minor changes (see Fig. 7 and Fig. 8). Now even small deviations of the anisotropy parameter χ from equilibrium have a major impact on equations and in consequence on behavior of the ASQUID. For small values of χ , at fixed value of the damping parameter Γ there are two possible solutions even for the first resonance. In symmetric SQUIDs this situation was present

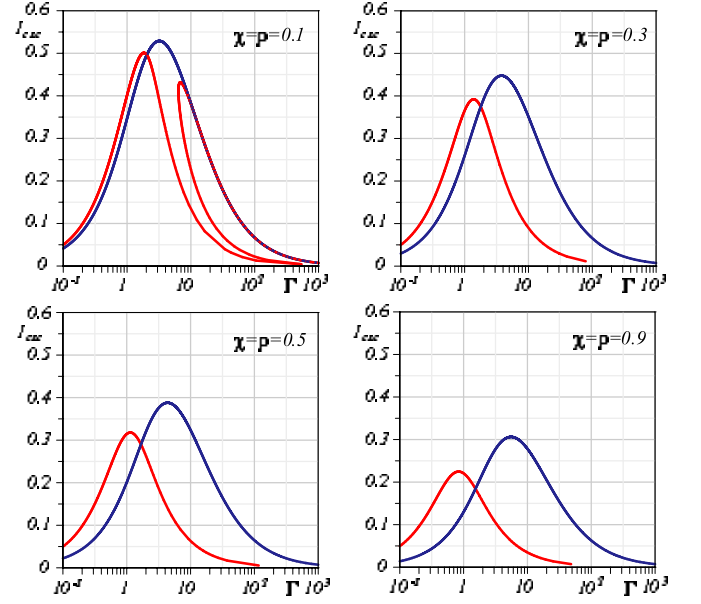


Figure 8: The normalized resonant current I_{exc} versus damping parameter Γ , for different values of the capacitance and dissipation anisotropy parameters χ (red) $= \rho$ (blue), first resonance ($n = 1$).

for higher resonances $n \geq 3$. Explanation of the latter comes from the fact that the resonant circuit oscillates at a frequency of ω_r , while Josephson current in the junctions oscillates at $n\omega_r$. In ASQUID we have three natural frequencies²⁷ $\omega_{1,2} = (L + C_{1,2})^{-1/2}$ and related $\omega_3^2 = \omega_1^2 + \omega_2^2$ which can be excited by the ac Josephson effect and converted through nonlinear interactions between junction and resonant circuit into dc current steps. Therefore introducing capacitance anisotropy we are able to create higher modes multivalued behavior of the excess current even for the first resonance.

3. Symmetric case ($\chi = \epsilon = \rho = 0$) with phase shift ($\vartheta_{\pm} \neq 0$)

This case corresponds with a situation where different phase shift is present in the junctions of the interferometer and analysis is similar to one carried by Chesca.²⁹ The equations take form:

$$\alpha n \omega = \gamma_+ - J_n(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-), \quad (61)$$

$$\gamma_- = \frac{2}{\beta} \phi_c - J_n(\delta) \sin(\theta - \vartheta_+) \cos(\phi_c + \vartheta_-), \quad (62)$$

$$\delta \left(\frac{2}{\beta} - \omega^2 \right) = J_n^-(\delta) \sin(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-) \quad (63)$$

$$\alpha \delta \omega = J_n^+(\delta) \cos(\theta - \vartheta_+) \sin(\phi_c + \vartheta_-). \quad (64)$$

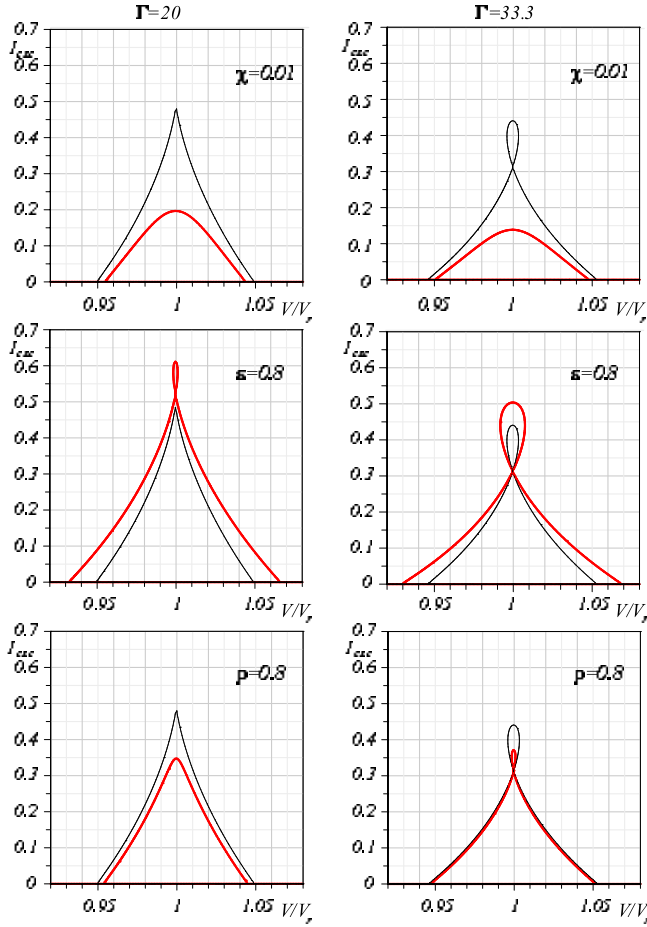


Figure 9: The normalized resonant current I_{exc} versus normalized voltage V/V_r , with different values of the anisotropy parameters, capacitance χ , the Josephson current ϵ , and dissipation ρ for second resonance ($n = 2$). Black curves refer to absence of anisotropy parameters.

We do not expect any changes in excess current - voltage characteristics. Rather, as it was pointed out by Chesca the difference between SQUIDs with various phase shifts can be visible only in magnetic field. In order to calculate excess current dependence on magnetic field we add squares of the equations (63) and (64). The resonant current is maximized when $\theta = \vartheta_+$ and we can write the solution in parametric form:

$$[I_{exc}; \sin(\pi\phi_e + \vartheta_-)] = \left[\frac{\delta^2}{2\Gamma n}; \frac{\delta}{\Gamma J_n^+(\delta)} \right]. \quad (65)$$

where δ is a dummy variable. Changing value of the parameter ϑ_- from 0 to $-\pi/2$ we have $0 - 0$ and $0 - \pi$ SQUID respectively. The shape of the surface describe current magnetic field dependence (see Fig. 10) remains unchanged but is translated by a vector $[0; -\vartheta_-]$ along ϕ_e axis.

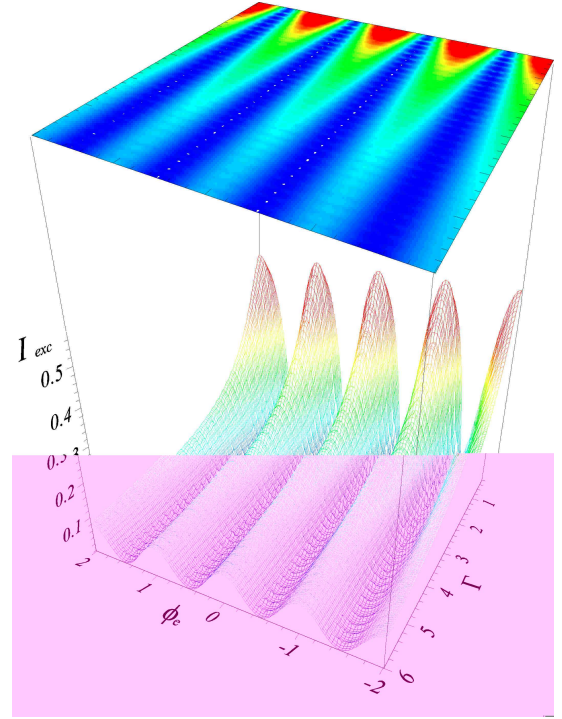


Figure 10: Excess current I_{exc} versus magnetic field ϕ_e (first resonance, $n = 1$) characteristic for different values of the damping parameter Γ for $0 - \pi$ interferometer.

V. DISCUSSION

The resonances in SQUIDs are investigated theoretically with several asymmetries: Josephson current ϵ , dissipation ρ and capacitance χ . In real devices it is impossible to have an ideal interferometer free of imperfections. In practice various deviations of the interferometer parameters from average values can occur together and mutually conceal each other. At this stage we have to separate discussion related to low- and high- T_C SQUIDs. In the former case, experimentally, we are able to control asymmetry of dissipative parameter ρ adding a parallel resistor to the junction but it is difficult to change the Josephson current independently from the capacitance. To produce the asymmetry of the Josephson current in the interferometer we can change the area of the junction A or thickness of the barrier d . Parallel-plate capacitor with area A of the plates and space d between them has the capacitance equal $C = \epsilon_r \epsilon_0 A/d$ for $A \gg d^2$, where ϵ_r is the relative dielectric constant of the interlayer dielectric and ϵ_0 is the vacuum electric constant. On the other hand the critical current can be written as $I_C = j_C A$ where j_C is the critical current density. These two simple relations imply that varying area ΔA of the junction in the interferometer we change both capacitance and critical current proportionally $\Delta I_C \sim \Delta C$ at the same time. When no further resistor is added to the junctions not only capacitance and Josephson current are related. From Ambegaokar-Baratoff³⁶ formula

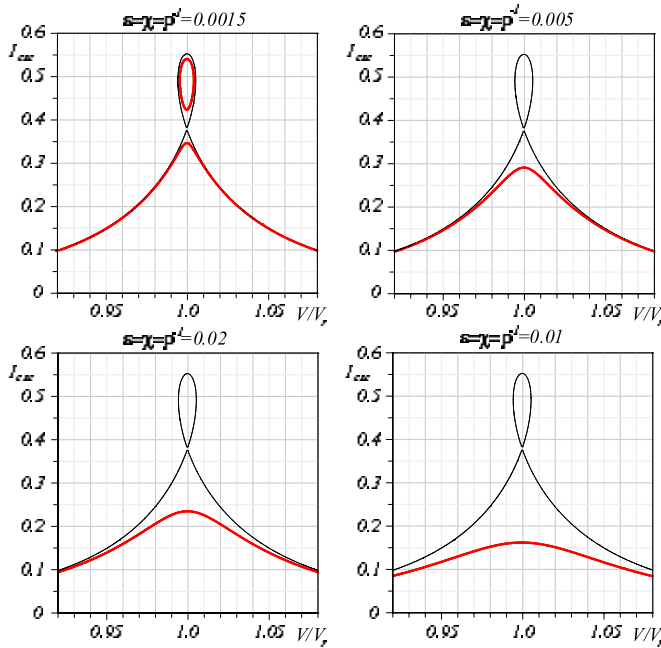


Figure 11: Current voltage characteristics ($I_{exc} - V/V_T$) for several asymmetric configurations of the SQUIDs related to the Ambegaokar-Baratoff formula ($\Delta I_C \sim \Delta C \sim \Delta R^{-1}$) for changes of the junction area ΔA , first resonance $n = 1$, $\Gamma = 20$, $\beta = 0.1$.

we know that the product $I_C R_N$, where R_N is the resistance in normal state, has an invariant value which depends only on the material in fixed temperature. Thus changing the value of the Josephson current we alter the resistance of the junction. Recapitulating these rather simple considerations we can introduce asymmetry in the Josephson current changing the area of the junction ($\Delta I_C \sim \Delta C \sim \Delta R^{-1}$). Setting parallel resistor we can control value of the resistance and vary dissipative parameter independently from the current asymmetry. We can also imagine junctions with different thicknesses of the barrier but technologically this case is difficult to achieve thus we do not consider it. In experiments with ASQUID both technically reached asymmetric cases do not differ very much because of the capacitance anisotropy. As we see from Fig. 11 the biggest impact on the maximum value of the resonant current has the anisotropy of the capacitance. Even small changes of χ can decrease excess current almost to zero.

The situation changes completely when high- T_C SQUIDs are considered. On one hand, the probability to find junction parameter asymmetries is particularly high, because high- T_C junctions are intrinsically affected by defects, as for instance faceting and/or oxygen vacancies inside the barrier. Moreover, up to now, the charge transport process is not completely understood, although various hypothesis have been proposed,^{37,38,41} and other recent experiments are still in progress.^{18,40} In particular, the simple rule $I_C R_N = \text{const}$ valid for low- T_C SQUIDs does not apply in the case of high- T_C interferometers

typically used in applications, based on the symmetric bicrystal c-axis [001] devices, and changing one single parameter is now possible. In such interferometers, $I_C R_N$ is proportional to the critical current density J_C at low values and stays roughly constant at high- J_C values.^{42,43} Moreover, HTS junctions are intrinsically shunted and SQUIDs are fabricated with no additional shunt resistor. As a consequence, the way to fabricate HTS SQUIDs with symmetric junctions is probably to reduce junctions' widths, limiting the effect of the interface defects. In all other cases, asymmetries will be very probable and our analysis could be relevant to understand the presence of resonance steps.

Different approach is necessary in the case of asymmetric [001] or [100] HTS bicrystal junctions, where the relation $I_C R_N$ seems to be similar to the one of low- T_C systems⁴³ and the necessity to account for effects of a non-conventional symmetry of the order parameter forces to include also the phase asymmetries in studying dynamical states in HTS interferometers. Finally, also the inclusion of the second harmonic term in the Josephson current in order to account for experimental results^{44,45,46} is mandatory. This will be the argument of a separate paper, and the possibility to deal with one single asymmetric parameter is now eventual. Moreover, a non conventional symmetry of the order parameter forces to include also phase asymmetries in studying dynamic states in high- T_C interferometers. In this frame the calculations derived in the present paper allow to investigate SQUID dynamics in both low- and high- T_C asymmetric devices.

VI. SUMMARY

In this paper we have presented a detailed theoretical study of the resonances in the asymmetric superconducting quantum interference device. Analytical approach revealed the nature of the resonances in the presence of several asymmetries: Josephson current ϵ , capacitances χ and dissipation ρ . Also we were able to derive magnetic field dependence of the excess current in presence of the magnetic field and phase shift. Our calculations imply that deviations of the capacitances from the average value in SQUID have profound impact on physics of the system. We have found that our theory can be useful to determine asymmetry parameters present in lightly damped ASQUIDs. Especially for SQUIDs produced from HTS materials where deviations from average values are practically inevitable our considerations are very helpful.

Acknowledgments

Authors would like to thank Prof. Antonio Barone and Dr. Ciro Nappi for a lot of fruitful discussions. This work was supported by the TRN "DeQUACS".

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